SE c-Scheme choice based (R-19 'C' Scheme) - Summer 2025 ECS

02-6-25

Time: 3 hour **Max. Marks: 80**

Note: 1) Question 1 is compulsory.
2) Attempt any 3 questions from Question 2 to Question 6
3) Figures to the right indicate full marks.

1.(a) Find the Laplace Transform of $f(t) = \int_0^t e^{-3u} sin4u \, du$. (5)

(b) If $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$, find the eigen values of $A^2 - 2A + I$. (5)

(c) Find half-range sine series for $f(x) = \begin{cases} 1, & 0 < x < a/2 \\ -1, & \frac{a}{2} < x < a \end{cases}$. (5)

(d) Find the constants a, b, c, d, e if $f(z) = (ax^4 + bx^2y^2 + dx^2 + cy^4 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic. (5)

2.(a) Evaluate $\int_0^\infty \frac{cosat - cosbt}{t} dt$ using Laplace Transform. (6)

(b) Show that the function $v = (x^4 - 6x^2y^2 + y^4) + (x^2 - y^2) + 2xy$ is harmonic and find the corresponding analytic function f(z) in terms of z. (6)

(c) Find the Fourier Series for $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$, $x \in [0,2]$. (8)

3.(a) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$. (6)

(b) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational. (6)

(c) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ hence find A^{-1}. (8)

4.(a) Use Stoke's Theorem to evaluate $\int \vec{F} . \vec{dr}$ where $\vec{F} = (2x - y)i - yz^2j - y^2zk$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = a^2, lying\ above\ xy\ plane.$ (6)

(b) Find the inverse Laplace Transform of $\frac{s+2}{s^2(s+3)}$. (6)

(c) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming and diagonal matrix. (8)

5.(a) Find the Fourier Series for $f(x) = x^2, -\pi \le x \le \pi$. (6)

(b) Find $L\{cosht \int_0^t e^u \, coshu \, du\}$ (6)

(c) Find $L^{-}\{\frac{1}{(s-a)(s+a)^2}\}$ using Convolution Theorem. (8)

6.(a) Evaluate by Green's theorem $\int (x^2 - y)dx + (y^2 + x)dy)$ over the closed curve C of the region bounded by $y = 4$ and $y = x^2$. (6)

(b) Find the inverse Laplace Transform of $log\left(1 + \frac{a^2}{s^2}\right)$. . (6)

(c) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (8)
